Learning Goal Check!

Prove the following identities

$$\sec x = \frac{2\cdot \sin x}{\sin 2x}$$

$$= \frac{1}{\cos x}$$

$$\frac{2\cancel{\sin x}}{2\cancel{\sin x}\cos x}$$

$$\frac{1}{\cos x}$$

$$2\cos^2 x = 3\cos^2 x + \sin^2 x - 1$$

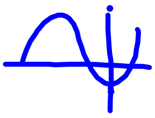
$$3\cos^2 x - (1 - \sin^2 x)$$

$$3\cos^2 x - \cos^2 x$$

$$2\cos^2 x$$

Homework Questions?

5c) $(\sin x + 1)(\cos x) = 0$



$\sin x + 1 = 0$

$\Rightarrow \sin x = -1$

$x = \sin^{-1}(-1)$

$= 180 + 90$

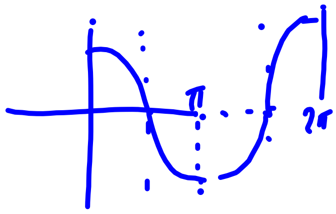
$= 270°$

$\left(\frac{3\pi}{2}\right)$

$\cos x = 0$

$x = 90°$

$x = 270°$



$x \in \{90°, 270°\}$

6c) $(2\cos x + \sqrt{3})\sin x = 0$ $\qquad \underline{0 \le x \le 2\pi}$

$2\cos x + \sqrt{3} = 0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

OR

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\sin x = 0$

$x = 0$

or

$x = \pi$

or

$x = 2\pi$

$x \in \{0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, 2\pi\}$

7a
e

7a) $2\cos^2 x + \cos x - 1 = 0$

Let $A = \cos x$

$2A^2 + A - 1 = 0$

$(2A-1)(A+1) = 0$

$A = \frac{1}{2}$ $\qquad\qquad$ $A = -1$

$\cos x = \frac{1}{2}$ $\qquad\qquad$ $\cos x = -1$

$x = \frac{\pi}{3}$ $\qquad\qquad$ $x = \pi$

or

$x = 2\pi - \frac{\pi}{3}$

$= \frac{5\pi}{3}$

$x \in \left\{ \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$

7e) $3\tan^2 x - 2\tan x = 1$

Let $A = \tan x$

$3\tan^2 x - 2\tan x - 1 = 0$

$3A^2 - 2A - 1 = 0$

$(3A+1)(A-1) = 0$

$A = -\frac{1}{3}$ $\qquad\qquad$ $\tan x = 1$ $\qquad$ $\frac{\pi}{4}$ $\qquad$ T | A

$\tan x = -\frac{1}{3}$

$x = \frac{\pi}{4}$

or

$x = \pi + \frac{\pi}{4}$

$= \frac{5\pi}{4}$

ran $\beta = \tan^{-1}\left(\frac{1}{3}\right)$

$\doteq 0.32$

$x = \pi - 0.32$

$\doteq 2.82$

or

$x = 2\pi - 0.32$

$\doteq 5.96$

8 d) $2\cot x + \sec^2 x = 0$

$\vdots$

f) $3\tan^3 x - \tan x = 0$

$(\tan x)(3\tan^2 x - 1) = 0$

$\tan x = 0$

$x = 0$

or

$x = \pi$

or

$x = 2\pi$

$3\tan^2 x - 1 = 0$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}}$

$\beta = \frac{\pi}{6}$

$x = \frac{\pi}{6}$

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$x \in \left\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi\right\}$

9d) $-2\cos 2x = 2\sin x$

$-2(1-2\sin^2 x) = 2\sin x$

$-2+4\sin^2 x = 2\sin x$

$-1+2\sin^2 x = \sin x$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\sin x = 1$

$x = \frac{\pi}{2}$

$x \in \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

14) $6\sin^2 x = 17\cos x + 11$

$6(1-\cos^2 x) = 17\cos x + 11$

$6 - 6\cos^2 x = 17\cos x + 11$

$0 = 6\cos^2 x + 17\cos x + 5$

$0 = (3\cos x + 1)(2\cos x + 5)$

$\swarrow \qquad\qquad \Downarrow$

$3\cos x + 1 = 0$

$\cos x = \frac{-1}{3}$

$2\cos x + 5 = 0$

$\cos x = \frac{-5}{2}$

S | A
T | C

$\beta = \cos^{-1}\left(\frac{1}{3}\right)$

$\doteq 1.23$

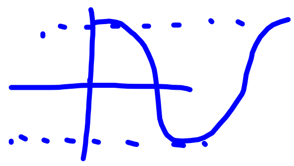
$x = \pi - 1.23 \doteq 1.91$

$x = \pi + 1.23 \doteq 4.37$

$\beta = \cos^{-1}\left(\frac{5}{2}\right)$

$\doteq$ NOT valid

i.e. no solution!



17.) $\dfrac{\tan x}{1-\tan x} - \dfrac{\cot x}{1-\cot x}$

$= \dfrac{\tan x}{1-\tan x} - \dfrac{\frac{1}{\tan x}}{1-\frac{1}{\tan x}}$ $\quad 1 = \dfrac{\tan x}{\tan x}$

$= \dfrac{\tan x}{1-\tan x} - \dfrac{\frac{1}{\tan x}}{\frac{\tan x - 1}{\tan x}}$

$= \dfrac{\tan x}{1-\tan x} - \dfrac{1}{\cancel{\tan x}} \times \dfrac{\cancel{\tan x}}{\tan x - 1}$

$= \dfrac{\tan x}{1-\tan x} - \dfrac{1}{\tan x - 1}$ $\qquad \dfrac{1}{\underset{-(1-\tan x)}{\tan x - 1}} = \dfrac{-1}{1-\tan x}$

$= \dfrac{\tan x}{1-\tan x} + \dfrac{1}{1-\tan x}$

$= \dfrac{\tan x + 1}{1-\tan x(1)}$ $\qquad \tan a = 1$

$\tan(x+a)$ $\quad 0 \le a \le 2\pi$

$= \dfrac{\tan x + \tan a}{1 - \tan x \tan a}$

$\therefore \tan a = 1$

$a = \dfrac{\pi}{4}$

or

$a = \dfrac{5\pi}{4}$

Unit 7 - Stuff You Should Know!

- Trig Identities!
  - > Must know these (no cheat sheet)
  - > Solve identities using LS=RS
  - > Use counter example to disprove
- Compound Angle & Double Angle Formulas
  - > Used to find nice answers to "non-nice angles"
  - > Also used in simplifying formulas
    - – working backwards
    - – expand and simplify
- Linear and Quadratic Trig Functions
  - > Solve for the angle(s)
    - – Use let statements to simplify and use previous knowledge
    - – Watch out for the domain restriction
  - > Generally, 2 or 4 answers

Homework:

pg 440 #2, 3d, 4a, 5ab, 6a,

8[1], 9[1], 10b, 12abc[2]

DO 12 c LAST.

[1]as usual, set up LS/RS

[2]for 12c, although using the quadratic formula yields the solution, the final simplification process is difficult. Instead, expand the LS, then “factor by grouping.”