Homework Questions?

1ec
2ce
3bef
9
12c
16
17

1c) $1-2\sin^2(3x)$

$= \cos 2(3x)$

$= \cos 6x$

$\cos 2x = 1-2\sin^2 x$

e) $4\sin\theta\cos\theta$

$= 2(2\sin\theta\cos\theta)$

$= 2\sin 2\theta$

$\sin 2x = 2\sin \underline{x} \cos \underline{x}$

2c) $2\sin\underline{\frac{\pi}{12}}\cos\underline{\frac{\pi}{12}}$

$= \sin 2(\frac{\pi}{12})$

$= \sin\frac{\pi}{6}$

$= \frac{1}{2}$

e) $1-2\sin^2\frac{3\pi}{8}$

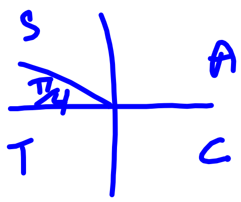
$= \cos 2(\frac{3\pi}{8})$

$= \cos\frac{6\pi}{8}$

$= \cos\frac{3\pi}{4}$

$= -\cos\frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2}$



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3b) $\cos 3x$

$= \cos 2\left(\frac{3}{2}x\right)$

$= 1-2\sin^2\left(\frac{3}{2}x\right)$ or $\cos^2\frac{3}{2}x - \sin^2\frac{3}{2}x$

or $2\cos^2\left(\frac{3}{2}x\right)-1$

e) $\sin x$

$= \sin 2\left(\frac{1}{2}x\right)$

$= 2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)$

f) $\tan 5\theta$

$= \tan 2\left(\frac{5}{2}\theta\right)$

$= \dfrac{2\tan\left(\frac{5}{2}\theta\right)}{1-\tan^2\left(\frac{5}{2}\theta\right)}$

9) $\sin\frac{\pi}{8}$

$\cos 2x = 1-2\sin^2 x$

$2\sin^2 x = 1-\cos 2x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\sin x = \pm\sqrt{\dfrac{1-\cos 2x}{2}}$

$\sin\frac{\pi}{8} = \pm\sqrt{\dfrac{1-\cos\frac{2\pi}{8}}{2}}$

$= \pm\sqrt{\dfrac{1-\cos\frac{\pi}{4}}{2}}$

$= \pm\sqrt{\dfrac{1-\frac{\sqrt{2}}{2}}{2}}$

$= \pm\sqrt{\dfrac{\frac{2-\sqrt{2}}{2}}{2}}$

$= \pm\sqrt{\dfrac{2-\sqrt{2}}{4}}$

$= + \dfrac{\sqrt{2-\sqrt{2}}}{2}$

12a) $\sin 3\theta = \sin(2\theta+\theta)$

$\sin(A+B)$

$= \sin A\cos B + \cos A\sin B$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= (2\sin\theta\cos\theta)(\cos\theta) + (\cos^2\theta - \sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$

$= 3\sin\theta\cos^2\theta - \sin^3\theta$

b) $\cos 3\theta = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= (\cos^2\theta - \sin^2\theta)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3\sin^2\theta\cos\theta$

16
17

16a) $\boxed{x = \tan 2A}$ $\boxed{y = \tan A}$

$$x = \tan 2A$$
$$x = \frac{2\tan A}{1-\tan^2 A}$$
$$x = \frac{2y}{1-y^2}$$

c) $x = \cos 2A$ $y = \csc A$

$\cos^{-1} x = \cos^{-1}(\cos 2A)$ $\csc^{-1} y = A$

$$\cos^{-1} x = 2A$$
$$\frac{\cos^{-1} x}{2} = A$$

$$\frac{\cos^{-1} x}{2} = \csc^{-1} y$$

17) $0 \le x \le 2\pi$

a) $\cos 2x = \sin x$

$$\cos 2x - \sin x = 0$$
$$(1 - 2\sin^2 x) - \sin x = 0$$
$$-2\sin^2 x - \sin x + 1 = 0$$
$$-2y^2 - y + 1 = 0$$
$$(2y^2 + y - 1) = 0$$
$$(2y-1)(y+1) = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$

Let $y = \sin x$

$\Rightarrow$ ① $2\sin x - 1 = 0$ or ② $\sin x + 1 = 0$

S | A
T | C

① $2\sin x - 1 = 0$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

② $\sin x + 1 = 0 \Rightarrow x = \frac{3\pi}{2}$
$$\sin x = -1$$

Lesson 7.04 - Proving Trig Identities

Learning Goals:

- I can prove any identity using previously established identities.
- To disprove a claim, I understand that I only require a counterexample to it.
- I can apply what I have learned in unfamiliar settings.

Verify the Identity:

$$\frac{\sin\theta}{\sin\theta + 1} = \frac{\csc x - 1}{\cot^2 x}$$

$$\tan^2 x(\csc x - 1)$$

$$\tan^2 x \csc x - \tan^2 x$$

$$\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sin x} - \frac{\sin^2 x}{}$$

See the posted cheat sheet for all of the Trig Ratios you need to know!

Example 1:

Prove: $$\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}=\frac{2}{\sin^2\theta}$$

LS

$= \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta}$

$= \frac{(1-\cos\theta)+(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{2}{1-\cos^2\theta}$

$= \frac{2}{\sin^2\theta}$

RS

$= \frac{2}{\sin^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

LS = RS ✓

Example 2:

Show that $\cos(2\theta) = 2\cos\theta$ is not an identity

when $\theta = 0$

$\cos(2(0))$ $\quad\quad$ $2\cos(0)$

$= \cos(0)$ $\quad\quad$ $= 2(1)$

$= 1$ $\quad\quad$ $= 2$

$\therefore$ by counter example

$\cos 2\theta \neq 2\cos\theta$

Example 3:

Use the compound angle identity to prove that $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

LS

$= \sin\left(\frac{\pi}{2} - \theta\right)$

$= \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$

$= (1)\cos\theta - (0)\sin\theta$

$= \cos\theta$

RS

$\cos\theta$

LS = RS ✓

Homework:

Today: pg. 417--418 #1, 5ac, 8 ,9abc, 17

Tomorrow: pg 418 #10abce, 11bdgjl