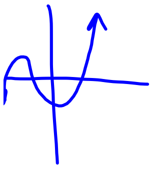
Questions from Homework?

pg 189

2 Assume it is cubic



$f(x) = a(x+4)(x+2)(x-2)$ $(0,-16)$

$-16 = a(4)(2)(-2)$

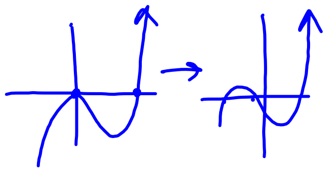
$-16 = -16a$

$1 = a$

$\therefore f(x) = (x+4)(x+2)(x-2)$

6,7. 4) $g(x) = x^3 - 4x^2$

$= x^2(x-4)$



6) $2x - 1 = 0$

$x = \frac{1}{2}$

$$\begin{array}{c|cccc} \frac{1}{2} & 6 & 1 & -12 & 5 \\ & & 3 & 2 & -5 \\ \hline & 6 & 4 & -10 & 0 \end{array}$$

$\therefore$ remainder $= 0$

by factor theorem $2x-1$ is a factor!

8) $x^4 + 3x^3 - 9x^2 + 6 = (D(x))(x^3 - 2x^2 + x - 5) + 31$

$x^4 + 3x^3 - 9x^2 - 25 = (D(x))(x^3 - 2x^2 + x - 5)$

$(x+5)$

$\dfrac{x^4 + 3x^3 - 9x^2 - 25}{x^3 - 2x^2 + x - 5} = D(x)$

$$\begin{array}{r|l} & x + 5 \\ \hline x^3 - 2x^2 + x - 5 & x^4 + 3x^3 - 9x^2 + 0x - 25 \\ & x^4 - 2x^3 + x^2 - 5x \\ \hline & 5x^3 - 10x^2 + 5x - 25 \\ & 5x^3 - 10x^2 + 5x - 25 \\ \hline & 0 \end{array}$$

Reverse synthetic division!

$$\begin{array}{c|ccccc} -5 & \boxed{1 \quad 3 \quad -9 \quad 0 \quad 6} \\ & & & & & 25 \\ \hline & \boxed{1 \quad -2 \quad 1 \quad -5} & \boxed{31} \end{array}$$

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0$

Lesson 4.01 - Solving Polynomial Equations Part 1

Learning Goals:

- I can solve polynomial equations

$$
\begin{gathered}
ax^2 + bx = c \\
4a^2x^2 + 4abx = 4ac \\
4a^2x^2 + 4abx + b^2 = b^2 + 4ac \\
(2ax + b)^2 = b^2 + 4ac \\
2ax + b = \sqrt{b^2 + 4ac}.
\end{gathered}
$$

Example 1:

Find the point(s) of intersection of these polynomial functions algebraically

① $y = 2x^4 + x^3 + x^2 - 7x - 20$

② $y = x^4 - 3x^3 - x^2 + 6x + 10$

① = ②

$2x^4 + x^3 + x^2 - 7x - 20 = x^4 - 3x^3 - x^2 + 6x + 10$

$1x^4 + 4x^3 + 2x^2 - 13x - 30 = 0$

factor this!

by factor theorem $x-2$

2	1	4	2	-13	-30
		2	12	28	30
	1	6	14	15	0

$(x-2)(x^3 + 6x^2 + 14x + 15) = 0$

by factor theorem $x+3$

-3	1	6	14	15
		-3	-9	-15
	1	3	5	0

$(x-2)(x+3)(x^2 + 3x + 5) = 0$

but $3^2 - 4(1)(5)$
$= 9 - 20$
$= -11$

$\therefore$ no real roots!

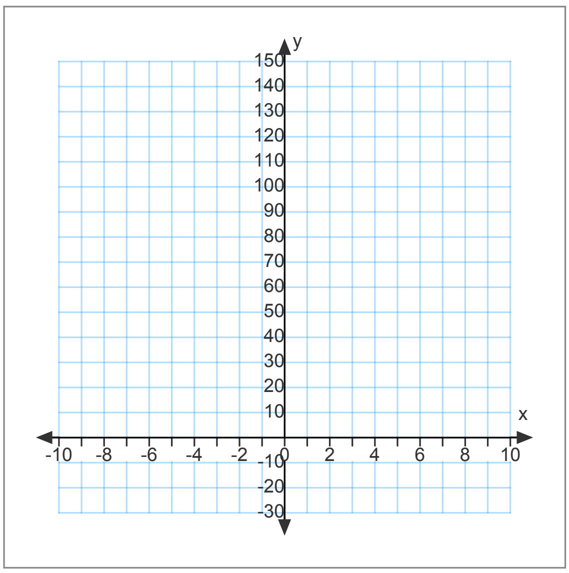
$\therefore (x+3)(x-2)(x^2+3x+5) = 0$

$\therefore$ either

$x+3 = 0$
$x = -3$

OR

$x-2 = 0$
$x = 2$



Example 2: Does 2 = 1?

Step 1. Let a and b be equal. Note: assume a and b are not zero.

Step 2. Multiply the equation by a.

Step 3. Subtract the equation by b^2.

Step 4. Factor the equation.

Step 5. Divide the equation by (a - b).

Step 6. Remember Step 1?

Step 7. Divide the equation by a.

1. $a = b$
2. $a^2 = ab$
3. $a^2 - b^2 = ab - b^2$
4. $(a-b)(a+b) = b(a-b)$
5. $(a+b) = b$
6. $(b+b) = b$

   $2b = b$
7. $2 = 1$?

but! You divide by 0!

$a-b$ remember step 1?

$= a - a$

or

$b - b$

Below is an example on the left of where the above misconception has been demonstrated by students:

Solve: $x^2 - 4x = 0$	Solve: $x^2 - 4x = 0$
$x^2 = 4x$	$x(x-4) = 0$
divide by x	Either $x = 0$
$x = 4$	or
	$x = 4$

Example 2: Solve $x \in R$

a) $2x^3 - 240x = 0$

$2x(x^2 - 120) = 0$

Lets try

$b^2 - 4ac$

$0 - 4(1)(-120)$

$= 480$ ... so it does have real roots...

$2x(x^2 - 120) = 0$

Either $2x = 0$ $\Rightarrow x = 0$

OR $x^2 - 120 = 0$

$x^2 = 120$

$x = \pm\sqrt{120}$

$= \pm\sqrt{4}\sqrt{30}$

$= \pm 2\sqrt{30}$

$\therefore x = 0$ or

$x = 2\sqrt{30}$

or

$x = -2\sqrt{30}$

b) $(x+2)(10x^2 - 19x - 15) = 0$

$(-19)^2 - 4(10)(-15) > 0$

$\therefore$ it factors

961

$\sqrt{961} = 31$

$(x+2)(5x+3)(2x-5) = 0$

$x + 2 = 0$

$x = -2$

or

$5x + 3 = 0$

$x = \frac{-3}{5}$

or

$2x - 5 = 0$

$x = \frac{5}{2}$

Homework:

pg 204 #1, 2, 3, 5, 6.
*For #2 you do not have to verify using technology.
Also for #2d one of the roots is -3 (not 3).