Questions from Homework?

10
16
17

10) $ax^3 - x^2 + 2x + b = f(x)$

$a(1)^3 - 1^2 + 2(1) + b = 10$

$f(1) = 10$

$a - 1 + 2 + b = 10$

$a + b + 1 = 10$

$a + b = 9$

$f(2) = 51$

$a(2)^3 - (2)^2 + 2(2) + b = 51$

$8a - 4 + 4 + b = 51$

$8a + b = 51$

Sub/Elim.

$8a + b = 51$

$-\ a + b = 9$

$7a = 42$

$a = 6$

$a + b = 9$

$6 + b = 9$

$b = 3$

16) $f(x) = x^3 - 6x^2 + 3x + 10$ $\quad x^2 - x - 2 = (x-2)(x+1)$

Show

$f(2) = 0$

$f(-1) = 0$

17) $f(x) = (x+a)^5 + (x+c)^5 + (a-c)^5$ $\quad x+a$ is a factor

$f(-a) = (-a+a)^5 + (-a+c)^5 + (a-c)^5$

$= (-1(a-c))^5 + (a-c)^5$

$= (-1)^5(a-c)^5 + (a-c)^5$

$= -(a-c)^5 + (a-c)^5$

$= 0$

$\therefore$ by factor theorem $f(-a) = 0$ so $x+a$ is a factor!

Lesson 3.07 - Factoring Sum and Difference of Cubes

Learning Goals:

- I can determine a factor a sum or difference of cubes

$$\begin{aligned}(x-3)(x+3) &= x(x+3)-3(x+3)\\ &= x^2+3x-3x-9\\ &= x^2-9\end{aligned}$$

Example 1:

Using the Factor Theorem, factor fully, if possible

$f(x) = x^3 + 8$

$f(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$x^3 + 8$

$= (x+2)(x^2 - 2x + 4)$

$f(x) = 8x^3 - 27$

$f\left(\frac{3}{2}\right) = 0$

$$\begin{array}{r|rrrr} \frac{3}{2} & 8 & 0 & 0 & -27 \\ & & 12 & 18 & +27 \\ \hline & 8 & 12 & 18 & 0 \end{array}$$

$8x^3 - 27$

$= \left(x - \frac{3}{2}\right)(8x^2 + 12x + 18)$

Factor Formula for a Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor Formula for a Sum of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Example 2:

Using the appropriate "new" formula, factor fully, if possible:

$f(x) = 8x^3 - 27$

$= (2x)^3 - 3^3$

$= (2x-3)(4x^2+6x+9)$

$27x^3 + 125y^3$

$= (3x)^3 + (5y)^3$

$= (3x+5y)(9x^2-15xy+25y^2)$

The Factor Theorem can be applied to any expression.

However, it may be more difficult to use than if one recognizes the expression as a sum/difference of cubes. Hence, the following algorithm is suggested, from now on:

When required to factor:

1. Is the expression a sum/difference of cubes? If so, use the appropriate formula.
2. Otherwise, apply the Factor Theorem directly

Homework

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Are you factoring fully?

$216x^3 - 8$

$8(27x^3 - 1)$

$8((3x)^3 - 1^3)$