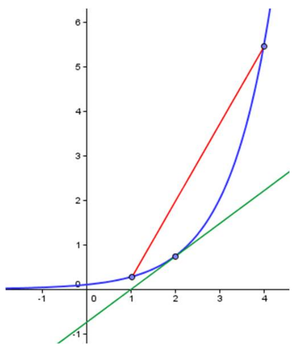
Lesson 2.02: Instantaneous Rate of Change Day 2

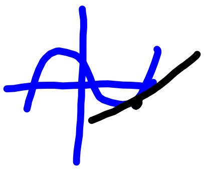
Learning Goals:

- I can determine the instantaneous rate of change using the difference quotient



Recall from Yesterday: To estimate the instantaneous rate of change of a function at point P, we can pick two points that are “close” to the point P and use the average rate of change to get an estimate.

$$\frac{\Delta y}{\Delta x} = \frac{f(x2) - f(x1))}{x2 - x1} \qquad \text{where } x1<x<x2$$



The Difference Quotient (First Principles)

$$m_{secant} = \frac{f(x+h) - f(x)}{(x+h) - (x)}$$

$$m_{tangent} = \frac{f(x+h) - f(x)}{x - x + h}$$

$$\boxed{m_{tangent} = \frac{f(x+h) - f(x)}{h}, \ h \to 0}$$

Example:

Determine the instantaneous rate of change to the curve $P(t)=100+30t+4t^2$ at $t=2.5$.

$$f(2.5)=100+30(2.5)+4(2.5)^2$$
$$=200$$

$(2.5, 200)$

$$m_{tangent}=\frac{f(2.5+h)-f(2.5)}{h}, h\to 0$$

$$=\frac{100+30(2.5+h)+4(2.5+h)^2-200}{h}, h\to 0$$

$$=\frac{100+75+30h+4(6.25+5h+h^2)-200}{h}, h\to 0$$

$$=\frac{\cancel{100}+\cancel{75}+30h+\cancel{25}+20h+4h^2\cancel{-200}}{h}, h\to 0$$

$$=\frac{50h+4h^2}{h}, h\to 0$$

$$=\frac{\cancel{h}(50+4h)}{\cancel{h}}, h\to 0$$

$$=50+\cancel{4h}^{\,0}, h\to 0$$

$$m_{tangent}=50$$

For each question, do NOT estimate the rate of change. Find the exact rate of change using the Difference Quotient...

Homework:

pp. 86-87 #2bc, 4ac, 5, 10* do not approximate π + Worksheet

Challenge: For the function y=1/x find the exact rate of change at x = 2

3π